1. a) Derive the finite element equation using the potential energy approach.  
b) Explain the various steps involved in solving a problem using finite element method.  

2. a) Derive an expression relating local and global coordinates.  
b) Explain the advantages of natural coordinates over other coordinates.  

3. The plane truss shown in figure 1 is composed of members having a square 10mm x 10mm cross section and modulus of elasticity $E = 90$ GPa. Compute the nodal displacements in the global coordinate system for the loads shown. Also, compute the axial stress in each element.  

4. a) State the properties of eigen vectors.  
b) Derive the element lumped mass matrix for a 2-dimensional beam element.  

5. How do you generate an iso-parametric quadrilateral element for $C^1$ continuity? Explain with example.  

6. a) How to evaluate the errors in Gauss quadrature numerical integration? Explain for two points method.  
b) Evaluate the $\int \left[2x^3 + 4x^2 + \frac{3}{(x+6)}\right] dx$ over the limits -1 and +1 using one point and two point Gauss quadrature methods.  

7. Prove how an isotropic axisymmetric solid element subjected to axisymmetric loading has effectively a 2-Dimensional state of stress.  

8. a) What are the current trends in finite element analysis software's? Explain the latest additive modules in ANSYS package.  
b) Explain methodology to consider the loads and boundary conditions over the domain for different types of loads using ANSYS package.
1. a) Explain the basic concept of FEM and list some of its advantages and applications. 
   b) Explain the principle of virtual work. [8+8]

2. a) Explain the natural coordinate system for one-, two- and three-dimensional elements. 
   b) Evaluate the natural coordinate \( \xi \), shape function \( N_1 \) and \( N_2 \) at point P shown in figure 1. 
   If \( q_1 = 0.075 \) mm and \( q_2 = -0.125 \) mm determine the value of displacement \( q \) at point P. [6+10]

3. a) Explain the difference between isoparametric, subparametric and superparametric elements. 
   b) Derive the stiffness matrix for a 3-D frame element. [8+8]

4. Find the nodal displacements and strain in element 2 for the truss shown in figure 2. 
   \( E=100 \) GPa; \( A=500 \) mm\(^2\). [16]

5. a) Discuss how the stiffness matrix can be evaluated for Isoparametric elements? 
   b) Distinguish between Lagrangian and Serendipity family elements. [8+8]
6. a) Explain the one-point Gaussian quadrature method for the numerical integration with suitable example.
b) What are the approximations and errors associated in one point Gaussian quadrature formula? Explain. [8+8]

7. The nodal coordinates for an axi-symmetric triangular element are 
\((r_1,z_1) = (20,10)\text{mm}, (r_2,z_2) = (40,10)\text{mm}, (r_3,z_3) = (30,50)\text{mm}.\)
Determine the Strain- Displacement matrix for this element. [16]

8. a) Explain different methods of mesh generation techniques.
b) Describe the ANSYS package and its uses in finite element analysis. [8+8]
1.a) Derive the finite element equation using the direct stiffness method.
b) Explain the various engineering applications of Finite element method and mention the field variable in each application. [8+8]

2.a) Derive the relation between natural and global coordinates.
b) In one-dimensional quadratic element, nodal displacement at $i^{th}$ node is $q_i = 6$ mm and $j^{th}$ node is $q_j = 8$ mm. The displacement at a point $P$ is given as $u = 6.25$ mm and the corresponding shape functions are $N_i = 1/4$ and $N_j = 1/6$. Find
i) $N_k$ and
ii) nodal displacement at $k^{th}$ node $q_k$. [8+8]

3. Calculate the nodal temperature using one dimensional heat transfer analysis in a fin (diameter=1 cm, length=8 cm). Using two elements and assuming the tip to be insulated. Base temperature=$100^\circ C$, Convection heat transfer coefficient $h=0.1\text{w/cm}^2\cdot\text{^0C}$, Thermal conductivity $K = 4 \text{ W/cm } ^0\text{C}$ and ambient temperature is $T_a = 30^\circ\text{C}$. Comment on the selection of the elements for obtaining accurate solution. [16]

4.a) What are the essential and natural boundary conditions in heat transfer problems?
b) Obtain the eigen values and eigen vectors for the cantilever beam of length 2m using consistent mass for translation dof with $E = 200\text{GPa}$, $\rho = 7500\text{kg/m}^3$. [6+10]

5.a) Discuss in detail the concept of Isoparametric Element?
b) Using Isoparametric concept, Formulate the Element Stiffness Matrix for a uni dimensional Two Noded element with constant cross sectional area ‘A’ and Modulus of Elasticity “E”. [8+8]

6.a) Discuss the importance of finite element modeling in solving the field problems.
b) How to store the large scale matrices? Explain with different methods. [8+8]

7.a) Derive the Jacobian matrix for 2-D axi-symmetric problems.
b) Explain the method to simplify the given domain using symmetric boundary conditions with suitable examples. [8+8]

8.a) How to generate the regions of mesh generation? Explain with examples.
b) Explain the methods for specifying the loads and boundary conditions in finite element modeling. [8+8]
1. a) Discuss about equilibrium, compatibility and convergence requirements related to finite element analysis.

b) Explain about simplex, complex and multiplex elements with respect to degree of freedom. [8+8]

2. a) Compute the shape functions at point Q shown in figure 1a using line coordinates.

b) For the triangular element shown in figure 1b obtain the shape functions at point P(2,2) within the element using area coordinates. [6+10]

3. Calculate the element stresses for the element shown in Figure 2 for plane stress and plane strain condition when nodal displacements are as given below:

\[ q_1 = 0, \ q_2 = 0, \ q_3 = 0.001\text{mm}, \ q_4 = 0.002\text{mm}, \ q_5 = -0.003\text{mm} \text{ and } q_6 = 0.002\text{mm} \]

\[ E = 200\text{GPa}, \ \nu = 0.25, \ \text{thickness} = 20\text{mm}. \] [16]
4.a) Derive the lumped mass matrix for one dimensional bar element.
b) Derive the stiffness matrix for a 2-dimensional frame element. [8+8]

5.a) Explain in detail how the element stiffness matrix and load vector are evaluated in isoparametric formulations.
b) Explain the concept of isoparametric elements and superparametric elements. [8+8]

6.a) Differentiate between Simpson's rule and Gauss quadrature.
b) Evaluate the Integral \( I = \int_{-1}^{1} (2x^3 + 5x^2 + 6) \, dx \) Using 3 Gaussian quadrature formula. [8+8]

7. Derive the stiffness matrix for 2-D axi-symmetric triangular element from the first principles. [16]

8.a) Explain different sub-structuring mesh generation techniques with suitable examples.
b) Compare different commercially available finite element software packages used for heat transfer analysis. [8+8]